[This question paper contains 4 printed pages.]

Your Roll No.............

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Sr. No. of Question Paper : 2819

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : B.A. (Prog.)

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. All questions carry equal marks.

1. (a) Let $S = \{x \in \mathbb{R} : x \geq 0\}$. Show in detail that the set S has lower bounds but no upper bounds. Show that inf S=0. Verify your answer.

(b) Define continuity of a real valued function at a point.

Show that the function defined as $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$

is continuous at $x = 3$.

(c) Let S be a non empty bounded set in $\mathbb{R}$. Let $a > 0$, and let $aS = \{as : s \in S\}$. Prove that $\inf aS = a \inf S$, $\sup aS = a \sup S$.

P.T.O.

(d) Test for convergence the series those nth term is $\left(\frac{\sqrt{n+1}-\sqrt{n-1}}{n}\right)$.

2. (a) A function f is defined by

$$f(x)=\begin{cases}\frac{1}{2}-x, & if\ 0<x<\frac{1}{2}\\ \frac{3}{2}-x, & if\ \frac{1}{2}\le x<1\end{cases}$$

Evaluate $\lim_{x\to\frac{1}{2}} f(x)$

(b) Define order completeness property of real numbers. State and prove Archimedean Property of real numbers.

(c) Show that the function f defined by $f(x)=x^3$ is uniformly continuous in the interval [0, 3].

(d) Prove that a necessary and sufficient condition for a monotonically increasing sequence to be convergent is that it is bounded above.

3. (a) State Cauchy's second Theorem on Limits. Prove that

$$\lim_{n\to\infty}\left[\frac{(2n)!}{(n!)^2}\right]^{1/n}=4$$

(b) Test for convergence the series whose nth term is $u_n=\frac{n^{n^2}}{(n+1)^{n^2}}$

(c) State Cauchy's general principle of convergence. Apply it to prove that the sequence $\langle a_n\rangle$ defined by

$$a_n=1+\frac{1}{4}+\frac{1}{7}+\dots+\frac{1}{3n-2}$$ is not convergent.

(d) Prove that a sequence of real numbers converges if and only if it is a Cauchy sequence.

4 (a) State D'Alembert's ratio test for the convergence of a positive term series. Use it to test for convergence the series $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n+1}$.

(b) A sequence $\langle a_n \rangle$ is defined as follows:

$$a_1 = 1, \quad a_{n+1} = \frac{4+3a_n}{3+2a_n}, \quad n \geq 1$$

Show that sequence $\langle a_n \rangle$ converges and find its limit.

(c) Show that the series

$$\sum_{n=1}^{\infty} \frac{2.4.6.....2n}{1.3.5.....(2n+1)} \text{ diverges.}$$

(d) Prove that if a function f is continuous on a closed and bounded interval $[a, b]$, then it is uniformly continuous on $[a, b]$.

5 (a) State Leibnitz test for convergence of an alternating series of real numbers. Apply it to test for convergence the series $\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + ...$

(b) Show that the function f defined by

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

is not integrable on any interval.

(c) Test for convergence and absolute convergence of the following series

$$\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + ...$$

(d) Show that the sequence defined by $\langle a_n \rangle = \left\langle \frac{n}{n+1} \right\rangle$ is a Cauchy sequence.

6 (a) Show that every Monotonic function on $[a, b]$ is integrable on $[a, b]$

P.T.O.

(b) Test the convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n\alpha}{n^p}$, p > 0. Is this series absolutely convergent.

(c) Show that the function $f(x) = [x]$, where $[x]$ denotes the greatest integer not greater than x, is integrable over [0, 3] and $\int_0^3 [x]dx = 3$

(d) Show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \tan\frac{1}{n}$ is convergent.